Zone Plate: A zone plate is an optical device based on the Fresnel's Theory of half-period zones. It consist of a plane parallel glass plate having concentric circles of radii proportional to the square root of the consecutive natural numbers 1, 2, 3 ... etc. Then even or odd order of annular spaces between the circles are made completely dark. Such a plate behaves like a convex lens and can produce image of a source of light on a screen placed at a suitable distance.

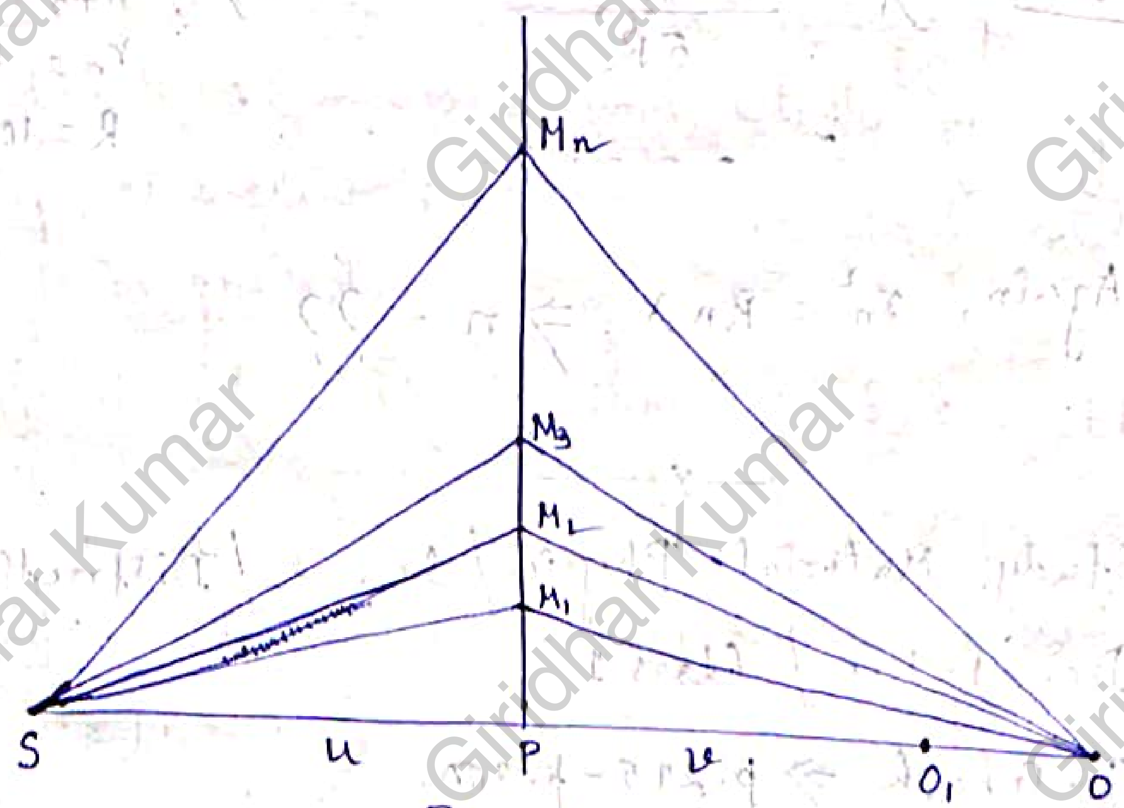

Fig. 1.

Let, $PM_n \rightarrow$ imaginary plane $\perp^r$ to the plane of the paper (2)

$S \rightarrow$ point source of light (having wavelength 'λ') in the plane of paper

$SP = u$, $PO = v$, $PM_n = r_n$ & $r_n << u$

We have to find out resultant disturbance at O due to spherical wavelets emitted by S.

The points $M_1, M_2, M_3 \ldots M_n$ on the plane (Fig. 1) are such that,

$$(SM_1 + M_1O) - SO = \frac{\lambda}{2}$$
$$(SM_2 + M_2O) - SO = \frac{2\lambda}{2}$$
$$- - - - - -$$
$$(SM_n + M_nO) - SO = \frac{n\lambda}{2} \longrightarrow (1)$$

The area of the circle of radius PM_1 on the plane is called first half period zone.

The area of the annular space between the circles of radii PM_1 and PM_2 is called the second half-period zone and so on.

Now, $SM_n^2 = r_n^2 + u^2$

$$\therefore SM_n = [r_n^2 + u^2]^{1/2} = u\left[1 + \frac{r_n^2}{u^2}\right]^{1/2}$$
$$= u\left[1 + \frac{r_n^2}{2u^2}\right] \longrightarrow (2)$$

Similarly, $M_nO^2 = r_n^2 + v^2$

$$\therefore M_nO = [r_n^2 + v^2]^{1/2}$$
$$= v\left[1 + \frac{r_n^2}{v^2}\right]^{1/2} = v\left[1 + \frac{r_n^2}{2v^2}\right] \rightarrow (3)$$

From (1), $(SM_n + M_nO) - SO = \frac{n\lambda}{2}$

$$\therefore u\left[1 + \frac{r_n^2}{2u^2}\right] + v\left[1 + \frac{r_n^2}{2v^2}\right] - (u+v) = \frac{n\lambda}{2}$$

(3)

$$\text{or, } u + \frac{r_n^2}{2u} + v + \frac{r_n^2}{2v} - u - v = \frac{n\lambda}{2}$$

$$\text{or, } \frac{r_n^2}{2}\left[\frac{1}{u} + \frac{1}{v}\right] = \frac{n\lambda}{2}$$

$$\text{or, } r_n^2\left[\frac{1}{u} + \frac{1}{v}\right] = n\lambda \longrightarrow (4)$$

From (4),

$$r_n^2\left[\frac{u+v}{uv}\right] = n\lambda$$

$$\text{or, } r_n^2 = \left(\frac{uv\lambda}{u+v}\right) n$$

$$\text{or, } r_n^2 = (\text{Constant})\, n$$

$$\therefore r_n \propto \sqrt{n} \longrightarrow (5)$$

Putting, $n = 1, 2, 3 \ldots$ etc. we get values of r_1, r_2, r_3 etc. Thus the radii of half period zones are proportional to the square root of natural numbers or zone numbers.

Now, if on a transparent plate circles are drawn with radii proportional to the square root of natural no.s 1, 2, 3 ... etc. and alternate zones are blackened, such a plate is called zone plate. The zone plate such constructed behaves as a convergent lens which will be discussed.

Area of n th. zone $= \pi (r_n^2 - r_{n-1}^2)$

$$= \left[\frac{\pi u v \lambda}{u+v}\right][n-n+1]$$

$$= \frac{\pi u v \lambda}{u+v} = \text{constant.}$$

Thus all the zones are of equal area. The numerical values of resultant amplitude $a_1, a_2 \dots a_n$ at 'O' due to secondary wavelets from 1st, 2nd etc. zone will have magnitudes in slightly decreasing order due to obliquity factor only. The wavelets from alternate zones differ in phase by π. Thus resultant amplitude at 'O' is given by $R = a_1 - a_2 + a_3 - a_4 + \dots$

If the 2nd, 4th., 6th. etc. zones are intercepted, the resultant amplitude at 'O' will be $R = a_1 + a_3 + a_5 + \dots$

This is many times greater than $\frac{a_1}{2}$, which is the resultant amplitude due to the wavelets from all the zones when none of them is made opaque. Thus the point 'O' will be, as a result, a point of maximum illumination.

Now from (4), $r_n^2\left[\frac{1}{u}+\frac{1}{v}\right]=n\lambda$

$$\text{or, } \frac{1}{u}+\frac{1}{v}=\frac{n\lambda}{r_n^2}=\frac{1}{f} \rightarrow (6)$$

This eq. is similar to the lens formula.

$f=\frac{r_n^2}{n\lambda}$ is called the principal focal length of the zone plate. Thus zone plate acts as a convergent lens with multiple foci for a particular wavelength, depending on the values of n and r_n.

<u>Multiple foci of zone plate</u>

If $u=\infty$, then $v=f=\frac{r_n^2}{n\lambda}$ i.e, image is formed at the principal focus on the axis of the zone plate (point 'O' in fig. 1). Let us consider another point O_1 along the axis of the zone plate, so that each exposed elements on the zone plate will contain three half period elements. The resultant amplitude at O_1 is,

$$R_1=(a_1-a_2+a_3)+(a_7-a_8+a_9)+\cdots$$

$$=\left(\frac{a_1}{2}+\frac{a_1}{2}-a_2+\frac{a_3}{2}+\frac{a_3}{2}\right)+\left(\frac{a_7}{2}+\frac{a_7}{2}-a_8+\frac{a_9}{2}+\frac{a_9}{2}\right)+\cdots$$

$$=\left[\frac{a_1}{2}+\left(\frac{a_1+a_3}{2}-a_2\right)+\frac{a_3}{2}\right]+\left[\frac{a_7}{2}+\left(\frac{a_7+a_9}{2}-a_8\right)+\frac{a_9}{2}\right]+\cdots$$

$$\simeq\frac{1}{2}\left[a_1+a_3+a_7+a_9+\cdots\right]$$

Thus the point O_1 is sufficiently bright.

O_1 represents the second focal point and the second focal length is $f_2 = \frac{r_n^2}{3n\lambda}$. The resultant amplitude at O_1 is less than that at O. So the intensity at O_1 is less than that at O. Similarly other foci occur at $f_3 = \frac{r_n^2}{5n\lambda}$, $f_4 = \frac{r_n^2}{7n\lambda}$ and so on. Intensity of successive foci decreases gradually.

It may be noted that maximum intensity occurs at those points for which the exposed elements of zone plate contain odd number of half period elements. If the clear space is occupied by even number of half period zones, they cancel in pairs and hence the intensity is zero.

Generalising the results, we can write,

$$f_m = \frac{r_n^2}{(2m-1)n\lambda} \longrightarrow (7)$$

where, m = 1, 2, 3, ...

Eq. (7) gives, $f_1 = \frac{r_n^2}{n\lambda}$

$$f_2 = \frac{r_n^2}{3n\lambda} = \frac{f_1}{3}$$

$$f_3 = \frac{r_n^2}{5n\lambda} = \frac{f_1}{5} \text{ and so on.}$$

Thus we see that a zone plate has multiple foci.